3.4. Conditional Semantics Revisited: Converse, Contrapositive, and Biconditional

1. Converse and Contrapositive. In constructing the semantic rule for conditionals, those cases where the antecedent is true seemed straightforward: if the antecedent and consequent are both true, the conditional is true (Valuation 1); and if the antecedent is true while the consequent is false, the conditional is false (Valuation 2).

Conditional Rule

	●	▲	(● → ▲)
V1	1	1	1
V2	1	0	0
V3	0	1	1
V4	0	0	1

More vexing were the cases where the antecedent is false (Valuations 3 and 4) – for there it seemed that the conditional staked no claim at all, and so could hardly be staking a *true* one.

Defense of the rule in these cases turned on the offensiveness of the alternative: making the conditional false here would leave it true only when both parts are true – and thus wrongly treat the conditional as equivalent to a conjunction.

But a critic might object that we've moved too fast here. For even if we can't have the conditional false in *both* these cases (for the reason just rehearsed), we still might have it false in just one or the other.

Of those two options, one is quickly seen to be unacceptable. If the conditional were false in the fourth valuation (while remaining true in the third), (● → ▲) would be equivalent to ▲ – that is, **the whole conditional would be logically equivalent to its consequent**.

☠ Alternative Conditional Rule #2 ☠

●	▲	(● → ▲)
1	**1**	**1**
1	**0**	**0**
0	**1**	**1**
0	**0**	**0**

That's certainly incorrect. It may, for example, be true that "If I won the lottery, then I'm a millionaire" but false that "I'm a millionaire". A conditional can be true when its consequent is false – contrary to what this rule claims.

The other alternative might seem more palatable: have the conditional true in Valuation 4, but false in Valuation 3.

☠ Alternative Conditional Rule #3 ☠

●	▲	(● → ▲)
1	1	1
1	0	0
0	1	**0**
0	0	**1**

But this alternative rule runs up against an observation made earlier: **order of parts** makes a semantic different in a conditional. For instance, if Pat is an adult, but it’s unclear whether Pat is a man or a woman, Sentence (1) is certainly true; but Sentence (2) might still false.

> (1) If Pat is a husband, then Pat’s married.
> (2) If Pat’s married, then Pat is a husband.

We say that (2) is the **converse** of (1). Our example illustrates that a conditional can be true when its converse is false – and hence that our semantic rule for the conditional had better not make it and its converse true in exactly the same situations, as a matter of law.

Our accepted semantic rule for the conditional gets this point right: “(P → Q)” can indeed be true while its converse, “(Q → P),” is false (the third valuation).

●	▲	(● → ▲)
1	1	1
1	0	0
0	1	1
0	0	1

P	Q	(P → Q)	(Q → P)
1	1	1	1
1	0	0	1
0	1	**1**	**0**
0	0	1	1

But Alternative Rule #3 doesn’t fare so well. Since this rule makes the conditional false whenever its two parts differ in value, “(P → Q)” and “(Q → P)” receive the same truth table.

☠ Alternative Conditional Rule #3 ☠

●	▲	(● → ▲)
1	1	1
1	0	0
0	1	0
0	0	1

☠ Result of Alternative Conditional Rule #3 ☠

P	Q	(P → Q)	(Q → P)
1	1	1	1
1	0	**0**	**0**
0	1	**0**	**0**
0	0	1	1

Not recognizing the semantic difference between a conditional and its converse, Alternative Rule #3 fails.

We note in passing that we can switch antecedent and consequent while retaining the same truth table – but only by negating both parts. "($\sim$Q $\rightarrow$ $\sim$P)" takes the same truth table as "(P $\rightarrow$ Q)".

P	Q	**(P → Q)**	~Q	~P	**(~Q → ~P)**
1	1	**1**	0	0	**1**
1	0	**0**	1	0	**0**
0	1	**1**	0	1	**1**
0	0	**1**	1	1	**1**

And that looks like the right result. English Sentences (3) and (4), for instance, do seem to stake the same claim.

(3) If Pat is a husband, then Pat's married.
(4) If Pat's not married, then Pat's not a husband.

"($\sim$Q $\rightarrow$ $\sim$P)" isn't the converse of "(P $\rightarrow$ Q)," but its **contrapositive**. The contrapositive of a conditional **is** logically equivalent to that conditional.

Converse of (P $\rightarrow$ Q): **(Q $\rightarrow$ P)**
Contrapositive of (P $\rightarrow$ Q): **($\sim$Q $\rightarrow$ $\sim$P)**

Recognizing that the converse is **not** equivalent to the conditional, but that the contrapositive is, our semantic rule makes correct predictions in a way that none of its alternatives can match.

2. Biconditionals. Since a conditional and its converse make two different claims – neither entailing the other – the conjunction of the two should make a stronger claim than either sentence alone.

Yet the truth table for that stronger sentence proves to be familiar.

P	Q	$(P \to Q)$	$(Q \to P)$	$\mathbf{((P \to Q) \land (Q \to P))}$
1	1	1	1	1
1	0	0	1	0
0	1	1	0	0
0	0	1	1	1

This is none other than Alternative #3, rejected above as the truth table for a simple conditional. An English example shows that this does indeed make a stronger claim: where it's settled that Rex is a man, we can make a claim for him that we couldn't earlier about ambiguous Pat.

> If Rex is a husband, then he's married; and if Rex is married, then he's a husband
>
> **P**: Rex is a husband **Q**: Rex is married
>
> $((P \to Q) \land (Q \to P))$

This sort of sentence – asserting a conditional and its converse – makes a **biconditional** claim.

A slight rephrasing brings out a more standard wording for the biconditional. By the above translation table we state "$(P \to Q)$" in English as "Rex is a husband only if he's married," and "$(Q \to P)$" as the inverted conditional "Rex is a husband if he's married".

The English conjunction of the two then reads as follows.

> Rex is a husband **if** he's married, and Rex is a husband **only if** he's married.

Deleted repetition yields the following traditional statement of the biconditional.

> Rex is a husband **if** ~~he’s married,~~ and ~~Rex is a husband~~ **only if** he’s married.
>
> Rex is a husband **if and only if** he’s married.

Certainly there are cases where it’s right to assert this stronger, two-way relation.

> (In a fixed volume of gas:) the temperature increases **if and only if** the pressure increases.
> A person is a bachelor **if and only if** he’s an unmarried adult male.
> An argument is valid **if and only if** it has no validity counterexamples.

And in some of these cases we may, in casual conversation, assert a mere conditional even though the stronger biconditional is clearly taken to be true.

> A person is a bachelor **if** he’s an unmarried adult male.
> An argument is valid **if** it has no validity counterexamples.

But in other cases we assert a mere conditional not due to any casualness of expression, but because **the converse is false**.

> If the score is 10-10, then the score is tied.
> If today is a Tuesday, then today is a weekday.

So a certain amount of context and background knowledge may be called for to make clear whether the speaker intends a conditional or a biconditional.

In many respects the biconditional mirrors the earlier case of the exclusive “or”: even though exclusive and inclusive disjunctions take quite different truth tables, conversational context might be needed to discern whether an “or” is intended inclusively or exclusively.

In fact the parallel goes further. For recall that when confronted with both exclusive and inclusive disjunctions, we had the option of complicating either our formal language, or our English-to-formal translations. That is: we could add a further connective to translate an exclusive "or" – say, the "⊕" symbol. Or we could keep the formal language lean in terms of connectives, at the expense of a longer formal translation: "$((P \vee Q) \wedge \sim(P \wedge Q))$". While we chose the second option, that was solely a matter of convenience and preference. The formal language would express the same sentence (with the same truth table) either way.

Likewise with biconditionals: we can choose to translate "P if and only if Q" in a more longwinded way, as "$((P \rightarrow Q) \wedge (Q \rightarrow P))$". Or we can introduce a further biconditional connective – traditionally, "$\leftrightarrow$" – and translate as "$(P \leftrightarrow Q)$".

Taking that second route, we would need to match the connective with construction and semantic rules, like so.

6. If ● and ▲ are formal sentences, then **(● ↔ ▲)** is a formal sentence.

Biconditional Rule

●	▲	(● ↔ ▲)
1	1	**1**
1	0	**0**
0	1	**0**
0	0	**1**

Since it's purely a matter of preference and convenience, in what follows we will feel free to appeal to this **biconditional sign** "↔" (pronounced "**bicon**") when translating English "if and only if" – or variations such as "**exactly on condition that**" and "**in just those cases where**". For as we'll see later in our discussion of expressive equivalence and expressive adequacy, we enjoy considerable latitude in language choice while retaining the full range of translation and truth tables.[1]

[1] See readings 3.X and 3.Y.

Summary: Converse, Contrapositive, and Biconditional

- "$(Q \to P)$" is the **converse** of "$(P \to Q)$".
- "$(\sim Q \to \sim P)$" is the **contrapositive** of "$(P \to Q)$". It is **logically equivalent** to "$(P \to Q)$".
- A sentence of the form "P **if and only if** Q" is a **biconditional**. Its formal translation, "$(P \leftrightarrow Q)$", is logically equivalent to "$((P \to Q) \wedge (Q \to P))$". The **biconditional sign** "$\leftrightarrow$" is called "**bicon**".
- Other English biconditional phrases are "**exactly on condition that**" and "**in just those cases where**".
- The semantic rule for the biconditional is as follows.

Biconditional Rule

●	▲	(● $\leftrightarrow$ ▲)
1	1	**1**
1	0	**0**
0	1	**0**
0	0	**1**